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Qualitative analysis of approximations of functions of a complex variable by Taylor series

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Abstract
The present work has been analysed graphically the approximations as Taylor series of functions of a complex variable, using the domain coloring. In this method each point is coloring according to the value of range. Functions of a complex variable have a property to associate one point of the plane other point of the plane, what describe them as vectorial functions from the complex plane to the complex plane. A complex function is called a holomorphic function in a region if is differentiable in all points of that region. Holomorphic functions can be described as power series.

Key words:
Complex Variables, Domain Coloring, Taylor Series.

Introduction

Functions of a complex variable have a problem to be represented, since them have four real variables involved: two in the domain and two in the range. So, there are complex numbers $z = x + iy$ in the domain that correponding to other complex numbers in the range, called $w = u + iv$, where $w = f(z)$.

Holomorphic functions can be represented by power series[1]. So, the objective of this paper is to analyze graphically some approximations of holomorphic functions using Taylor series.

Domain coloring is a method to study functions of a complex variable, and it was first used in the 1980s by Larry Crone and Hans Lundmark[2]. In this method each point is coloring about its norm and argument: according the distance between the point and the origin is given your opacity, and according your argument is given your tonality. In this study is adopted the referential described in the Image 1. To study the complex functions, each point z is coloring according to the value of $f(z)$, what enables a simultaneous representation of domain and range of these functions. In Image 2 (a) is possible to see the function $f(z) = \sin z$ represented by the domain coloring.

Results and Discussion

Taylor series of a real valued function $f(x)$ is the series $\sum_{n=0}^{\infty} \frac{f^{(n)}(a) \cdot (x-a)^n}{n!} = f(x)$, if $f(x)$ is infinitely differentiable at a. Fixing a natural number k, is possible to approximate that function as the polynomial described by the finite sum $\sum_{n=0}^{k} \frac{f^{(n)}(a) \cdot (x-a)^n}{n!} \approx f(x)$. Using complex variables there are the same process, where a is a complex constant and the independent variable is complex.

The present paper analyzed approximations of functions of a complex variable by Taylor series. So, the domain coloring has been used for this. Each approximation can be described graphically using that method, and then it's possible to see how the approximations are transforming the complex plane, and compare to the original function.

Is possible to see in the Image 2 the approximations of $f(z) = \sin z$ around the origin: $w = z - \frac{z^3}{3!}$ (b), $w = z - \frac{z^3}{3!} + \frac{z^5}{5!}$ (c) and $w = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!}$ (d). An interesting fact in these images is how the colors have become the same as the original function in larger regions, while k assume high values.

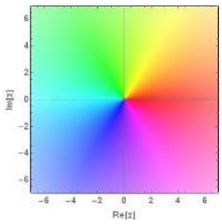

Image 1. Domain coloring referential.

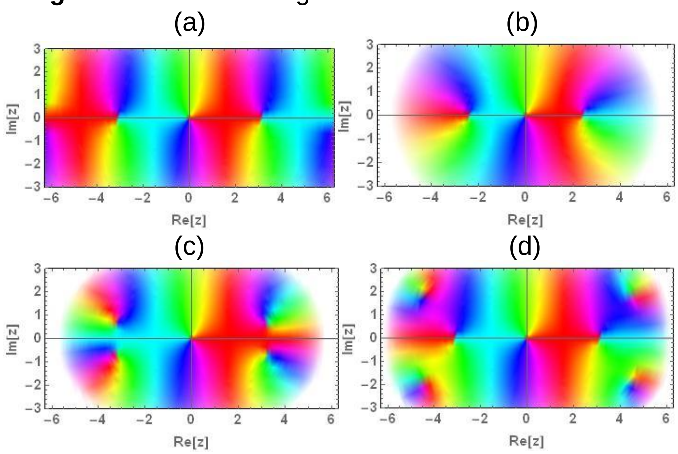

Image 2. Function $f(z) = \sin z$ by the domain coloring (a), and its Taylor polynomials when $k = 3$ (b), $k = 5$ (c) and $k = 7$ (d), around the origin.

Conclusions

Using these results is possible to see the regions of convergence of the Taylor polynomials in complex variables, that is, the regions where $\sum_{n=0}^{k} \frac{f^{(n)}(a) \cdot (z-a)^n}{n!} = f(z)$, for some k natural. Starting from this, the more difficult operations can be accomplished using the polynomials, sometimes simpler than the original functions.

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1 ÁVILA, G. S. S. *Funções de uma Variável Complexa*. Rio de Janeiro: Livros Técnicos e Científicos Editora S.A., 1974.
2 WEGERT, E. *Visual Complex Functions: An Introduction with Phase Portraits*. Springer Basel, 2012. p. 29. ISBN 9783034801799.