The great superconducting wave function

Guillermo G. Cabrera, Hiero S. De Paula*

Abstract
This work was developed as a guided study on the topic of superconductivity (SPC), quantum field theory (QFT) and their relation. We started with the basic phenomenology of SPC followed by a study of Ginzburg-Landau theory (GLT), its parameters and consequential categorization of SPC into types I and II where the solution proposed by Abrikosov was analysed thoroughly. Subsequently, in order to construct the analogy between SPC and the spontaneous symmetry breaking of the electromagnetic gauge, we studied concepts from the theory of scalar fields, from basic results, through gauge symmetries to the Abelian Higgs mode.

Superconductivity, Ginzburg-Landau theory, Spontaneuous Symmetry Breaking.

Introduction

Superconductivity is a phenomenon is characterized the by zero electrical resistance and expulsion of magnetic fields that occurs in certain types of materials when cooled below a critical temperature (CT). The GLT was one of the two remarkable theories (together with microscopic BSC theory) that arrived at an understanding of "conventional" SPC and made able for a categorization of superconductors into two types reffered as I and II. In GLT the free energy of a superconductor can be expanded near the phase transition in terms of an order parameter, a parameter that is zero for temperatures above the CT and non null below it. The free energy expansion of GLT has the same form as the lagrangian of a charged scalar field with gauge U(1) symmetry, thus, SPC can be seen from the perspective of quantum field theory as an ilustration of the Abelian Higgs mechanism, a case of spontaneous symmetry breaking. Our goal was to study the phenomenology of SPC and the details of its categorization through GLT and, with the development of the theory of scalar fields, study the analogy between SPC and the symmetry breaking of the electromagnetic gauge.

Results and Discussion

We started studying how SPC manifests itself by the phenomenon of zero resistivity and the emerging of the Meissner effect at temperatures below the CT. The behavior of the magnetic field inside the superconductor was analyzed and the london penetration depth obtained. The thermodynamics of SPC were studied in sequence in order to derive equations for the free energy at the phase transition and be able to understand the emergence of the mixed state where, at certain conditions, the flux penetrates the superconductor in quantized manner. From the GLT the order parameter was defined and the expansion of the free energy in terms of it introduced. The equations of GL were derived and the characteristic lenghts of the theory were obtained from them: the coherence length and again the london penetration depth. The categorization SPC types was then made possible by the GL parameter, which was later understood in more depth. The quantum of magnetic flux was also obtained from the equations and the their solution through the ansatz of Abrikosov was studied in detail giving better intuition on the parameters obtained. We turned then our attention to the study of field theory. We started with the derivation of the Klein-Gordon and Euler-Lagrange equations followed by understanding the consequence of symmetries through Noether's theorem. Electric charge conservation led us to the unitary group U(1), and the invariance of the action by local transformations forced the introduction of a gauge field leading to the scalar electrodynamics lagrangian. The symmetry breaking induced by the variation in sign of the mass term introduced the Abelian Goldstone mode in the case of a global transformation and the Abelian Higgs mode in the local one. Here the similarity between this lagrangian and the free energy expansion led us to an analogy between SPC and the Higgs mechanism. The GL equations could be derived as movement equations from this lagrangian, the order parameter which is the density of cooper pairs was identified with a scalar field and the emergence of SPC could be seen as a case of spontaneous symmetry breaking of the electromagnetic gauge.

Conclusions

We were able to arrive at the desired comprehension of the subject by the structure of the work done. The concepts of QFT were studied in more depth than it was planned and the analogy was successfuly constructed and understood.

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