

Image Processing and Analysis via Fuzzy Transform

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Abstract

The fuzzy transform is a mathematical tool that unifies the traditional concept of function transforms with fuzzy rule-based systems. In this project, we studied applications of the fuzzy transform for edge detection and compression of grayscale images. To this end, definition and mathematical properties of the fuzzy transform were studied. Finally, computational experiments were performed to quantitatively evaluate the studied approaches.

Key words:

Fuzzy sets, fuzzy transform, image processing and analysis.

Introduction

The F^d -transform of a function f of two variables with respect to fuzzy partitions A_1, \dots, A_n and B_1, \dots, B_n is an array $\mathbf{F}_{nm}^d[f] = (F_{kl}^d)$ of polynomials of two variables of order d . Given array of polynomials of order d , the inverse F^d -transform can be used to obtain an approximation of the original function f .

The fuzzy transforms exhibit many interesting approximation properties. For instance, we can approximate f and its derivatives using respectively the F^0 -transform and the F^1 -transform^{1,2}.

Results and Discussion

A digital grayscale image is interpreted as a real-valued function f from a $N \times M$ grid to the unit interval $[0,1]$, where 0 represents black and 1 represents white.

The $n \times m$ F^0 -transform, $\mathbf{F}_{nm}^0[f]$, of a grayscale image f of size $N \times M$ can be interpreted as a compressed version of f if $n < N$ and $m < M$ because the inverse F^0 -transform yields an approximation of the original image. In this case, the compression ratio is defined as the ratio between the size of the F^0 -transform and the size of the original image. Figure 1 shows a comparison between this method and a SVD based compression using the structural similarity index (SSIM)⁴.

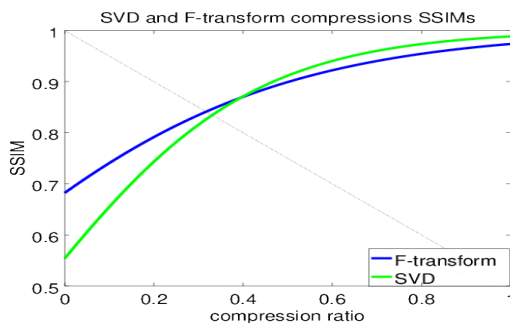


Figure 1. Comparison between the F^0 -transform and SVD compression.

The F^1 -transform provides an estimate of the gradient of a grayscale image². The gradient is used to obtain a binary image with one-pixel wide edges that

should represent the original image's edges³. We compared the edge detection capability of the fuzzy transform with different partitions and the morphological gradient³ using grayscale images from BSD300 image database⁵. Figure 2 shows the boxplot obtained using Pratt's figure of merit (FoM)³ as the measure of performance of an edge detector. Here, a greater value means a better performance.

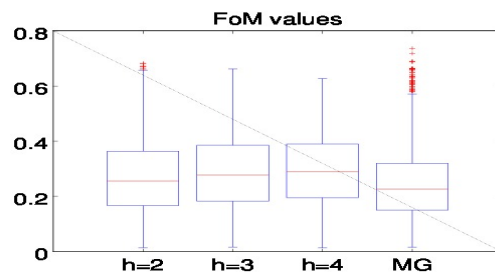


Figure 2. Boxplot representing the FoM values of the edge detector based on the fuzzy transform with h -uniform triangular fuzzy partitions² with $h = 2, 3, 4$ and the detector based on the morphological gradient (MG).

Conclusions

For image compression, the fuzzy transform outperformed the SVD-based approach for lower compression ratio.

The fuzzy transform edge detector, on average, proved to be effective for the greater values of h .

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