Metodologia de Pré-Condicionamento de Métodos Iterativos Utilizando Matriz Inversa com Precisão Simples. / Methodology of interative methods preconditicionated using the Inverse Matriz with single-precision.

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Abstract

The purpose of this project is to propose a precondicioner for linear systems with sparce matrices using the inverse matrix calculated with single-precision and compare the results. The importance of this study relies on the use of large space matrices in several mathematical, eletrical and engeering problems, and how precondicioners are essencial when making calculations with large matrices. The idea of using the single-precision in order to find the precondicioner is motivated by a smaller computacional effort when compared to the double one, and yet close enough to the answer.

Key words:

Matrix Optimization, Preconditioners, Iterative Methods.

Introduction

Many engeneering problems can be posed as a sparce matrix problems. It is common to use matrices with double precision in computacional calculations, due to a final sotution with more significative figures when compared to single precision. However the use of double precision requires more computational effort. In order to improve the efficiency of computational algorithms, it was opted to solve linear problems using iterative methods: using the very same matrix in single precision as a precondicioner to the double precision matrix.

Results and Discussion

The different methods of solving linear systems, such as Gauss elimitation, LU decomposition, LDLT, LDL, Cholesky decompostions and iterative methods like the Rank-1-Update, Steepest Descent and Conjugated Gradient Method were studied. The study was done in 3 stages: a theoretical analysis, the analysis of code developed using Mathematica and a final study of code in C++.

In order to compare the methods studied, we downloaded matrices from The University of Florida Sparse Matrix Collection, all matrices studied were square, positive definite and real. These caracteristics are suposed to make the code perform well once theoretical garantees were respected. Besides that, the function posix_time from the Boost library was used to get the time used in each operation/process.

The main code can be devided into 4 schemes. The first does the Cholesky decomposition of the matrix using double precision. The second in single precision. The thrid includes the conjugated gradient method, and informs the number of iterations used to find the inverse matrix. The last scheme calculates the linear system using both the preconditioned matrix and the inverse matrix calculated with the conjugated gradient method in single precision and the solution using Cholesky Decompostion directly.

Conclusions

The result demonstrated that the use of the inverse matrix in single-precision as a precondicioner was more effective than the use of no precondicioner for most cases. The diferences in the research results might be explained by size of the matrix, the disposition of the non-zeros in the matrix, convergence of the matrix and time to solve the linear sistem. This code has a better performance for large matrices, with at least 90 thousand equations.

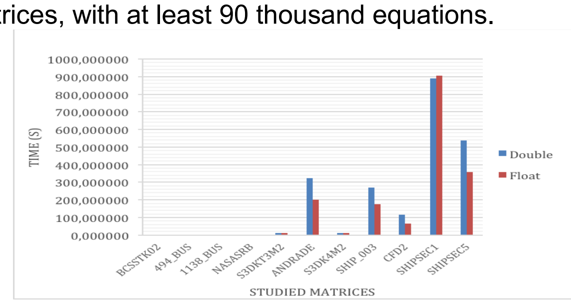

Image 1. Time to decompose the matrices studied in float and double using Cholesky decomposition.

Name	Number of Equations	Amount of non-zeros	Number of iterations CG	Residual Vector
bcsstk02	66	4356	2	8,040*10ˆ-10
494_bus	494	1666	3	5,267*10ˆ-10
1138_bus	1138	4054	3	4,657*10ˆ-10
nasasrb	54870	2677324	7	1,691*10ˆ-7
s3dkt3m2	90449	3686223	10	20,460
andrade	90450	7612986	3	5,771*10ˆ-10
s3dk4m2	98449	4427725	10	2,081
ship_003	121728	3777036	5	4,034*10ˆ-7
cfd2	123440	3085406	3	2,072*10ˆ-7
shipsec1	140874	3568176	4	2,879*10ˆ-9
shipsec5	179860	4598604	3	2,396*10ˆ-7

Chart 1. Matrix analysis performance using the Gradient Conjugated Method

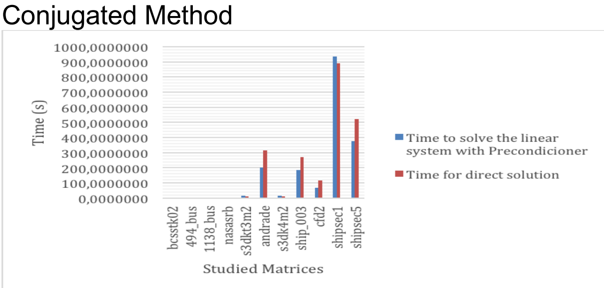

Image 2. Time to solve the linear system with precondioner versus the time to solve the linear system directly in double.

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