

Control Through the Network and Information Theory

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Abstract

Recent control projects have found a limitant factor to their development, the potential of communication between controller, sensors and actuators, a factor that is not taken into account in the classical controller design. This research aims to establish connections between aspects of communication theory and the stability of Markov jump linear systems.

Key words:

Markov Jump Linear Systems, Information Theory, Channel Entropy.

Introduction

Classical control theory usually studies dynamic systems interconnected through ideal communication channels, and communication theory studies the information exchange through imperfect channels. Typical phenomena from data transmission, such as packet dropouts, bandwidth constraints, delay, encoding etc. are normally disregarded in control designs. A new research area, named Networked Control Systems, is starting to bring together both control and communication theory.

There is a class of stochastic dynamic systems named Markov Jump Linear Systems (MJLS), which is particularly suited to model the events of sensor and/or actuator failures. Therefore, it is of particular interest to model transmission dropouts.

An MJLS is modeled through a set of finite subsystems, each represented by linear parameters. The determination of which subsystem is active at a given instant is carried out by the value of a random variable governed by a Markov chain with given transition probabilities. Stability for this class can be defined as second moment stability (SMS)¹

A common way to understand the potential of data compression and reliable transmission is in terms of Shannon's Information Theory, which establishes limits for source and channel coding.

Our main objective here is to find a relation between the dynamics of the individual linear systems representing each mode of the MJLS and the information generated by the associated Markov chain establishing if the MJLS is SMS.

Results and Discussion

In order to find a relation between the individual dynamics of each subsystem within the MJLS and the entropy of the associated Markov Chain, we decided to work with a discrete-time scalar system with two Markov modes and given transition probability matrix.

$$G : x_{(k+1)} = a_{(\theta k)} * x_{(k)} \quad (1)$$

$$P = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix} \quad (2)$$

The SMS stability can be checked by a set of linear matrix inequalities (LMIs). We can say that the system of

equation (1) is SMS if and only if, there exist $m_1 > 0$ and $m_2 > 0$ such that:

$$\begin{cases} a_1^2 * [p * m_1 + (1-p) * m_2] - m_1 < 0 \\ a_2^2 * [(1-q) * m_1 + q * m_2] - m_2 < 0 \end{cases} \quad (3)$$

Based on those constraints we could find the following necessary condition for SMS:

$$a_1^2 < \frac{1}{p}, a_2^2 < \frac{1}{q} \quad (4)$$

Manipulating the relations found in (4), we can write:

$$\begin{cases} \log_2 a_1 < -\frac{1}{2} \log_2(p) = I(p) \\ \log_2 a_2 < -\frac{1}{2} \log_2(q) = I(q) \end{cases} \quad (5)$$

where $I(x)$ denotes the information obtained by the observation of an event of probability x . This indicates that the information measures of the chain parameters are directly related to the maximum values of the system coefficients, so that stability can be ensured.

Conclusions

For a simple scalar, two-mode MJLS, we were able to establish interesting connections between Shannon's information of the associated Markov chain probabilities and the SMS stability. For future work, we aim to generalize those results to consider higher order dynamics and an arbitrary number of Markov modes.

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¹ Costa, O.L.V., Fragoso, M.D., Marques, R. P., Discrete-Time Markov Jump Linear Systems, **2005**, Springer.