

## Why do we need extra spacetime dimensions in bosonic string theory?

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### Abstract

The two main discoveries of early nineteenth century theoretical physics, namely, General Relativity and Quantum Mechanics, are known to be mutually inconsistent, and the usual quantization procedures fail when applied to the former. String theory, one of the few existing approaches to quantum gravity, predicts some weird results, such as extra spacetime dimensions. Here we show that in bosonic string theory, the first developed model of quantum strings, we need the underlying spacetime to be 26-dimensional for consistency reasons.

*Key words: quantum gravity, string theory, extra dimensions.*

### Introduction

General Relativity and Quantum Mechanics are known to be mutually inconsistent and classical quantization methods failed when applied to the former. String theory provides one viable way to a theory of Quantum Gravity, but is a constrained theory: we show that the exigency of consistency with special relativity fixes the dimension of the underlying spacetime in 26 (in the absence of supersymmetry).

### Results and Discussion

We follow the standard prescription of light-cone gauge quantization of the Nambu-Goto string action. Strings moving in spacetime describe a surface, called world-sheet. There is a notion of "area" of such a surface, coming from integration of the canonical volume form associated to the pullback of spacetime metric to the world-sheet: this is precisely Nambu-Goto action. Application of the Principle of Least Action shows that strings move in such a way as to minimize this area, like soap films in spacetime.

Equations of motion are hard to solve directly, but the action has gauge symmetries, corresponding to the possible reparameterizations of the world-sheet. A judicious choice of gauge, called the light-cone gauge, gives us wave equations for each component of the parameterization. The classical solution can then be written in terms of oscillation modes.

Quantization is carried out postulating usual commutation relations between position and momentum operators and then the oscillation modes become creation/annihilation operators. Working in the light-cone gauge has its price, however: we are not sure a priori that the theory is Lorentz invariant, i.e., consistent with special relativity. Thus we impose that quantum Lorentz charges satisfies Lorentz algebra and this turns out to be dependent of the spacetime dimension  $D$ , holding only for  $D=26$ . We also construct the

state space of the quantum open string, which have massless states associated to photons, as well as states with negative mass-squared, called tachyons.

### Conclusions

Consistency demands that spacetime is 26-dimensional in bosonic string theory. However, the existence of tachyons suggests that the vacuum of the theory is unstable, which is a big problem and show it cannot be true. It is a toy model, indeed: superstring theories avoid tachyons by imposing supersymmetry.

### Acknowledgement

This work was financially supported by CNPq/PIBIC scholarship.