

## Geometry of locomotion on highly viscous fluids

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### Abstract

On highly viscous fluids inertial forces are so small that swimming is not possible by means of the techniques we are used to. Nevertheless, locomotion is still possible by means of cyclic deformations that give rise to a geometric phase effect. The geometrical nature of the problem allows a gauge-theoretic approach to it, which we review in this project. In particular, we define the Stokes' connection, compute its curvature and revisit a classic calculation in the context of this framework.

*Key words: differential geometry, gauge theory, fluid dynamics*

### Introduction

At low Reynolds number inertial forces are negligible compared to viscous forces. Because of that, locomotion as we know it is not possible, inasmuch a body is not able to exert net forces and torques on the surrounding fluid.

On the other hand, a deformable body may be able to induce a net rigid motion by performing cyclic deformations which do not require external forces and torques.

This is the way according to which several microorganisms, such as a number of bacteria, are able to travel through the fluids in which they live.

Although the problem may be solved by a brute force approach without the techniques we have employed, gauge theories provide a framework with several advantages, one of them being the fact that the problem gains a clearer formulation with well-defined concepts.

What allows this framework to be used is the purely geometric nature of the problem: on highly viscous fluids time becomes just a parameter, momentum is instantly diffused and only the geometry of the sequence of deformations matters.

This allows one to take advantage of the methods of gauge theories.

### Results and Discussion

We consider the space of all possible configurations of a deformable body to be the space of all parameterizations of possible boundaries of the body, located and oriented in some manner. This allows us to define, in a natural way, a fiber bundle whose structure group is the group of rigid motions in Euclidean space. The base space of this bundle is the space of all possible shapes regardless of location and orientation while the total space is the configuration space of the body.

We then showed that it is possible to solve the problem of locomotion at low Reynolds number by introducing an appropriate connection, known as Stokes' connection, on this bundle.

After that, we computed the curvature of this connection and showed how the knowledge of the curvature allows one to compute the velocity acquired by the body when performing infinitesimal cyclic deformations. In this process we have reviewed a theorem which relates the curvature coefficients to the solutions to the Stokes' equations, with boundary conditions given by the infinitesimal deformations.

Finally, we have used this framework to study a classic problem which is the swimming of an inextensible sheet in a highly viscous fluid.

### Conclusions

Applying methods from gauge theories we revisited the problem of locomotion of deformable bodies in highly viscous fluids and showed how it can be formulated very naturally in terms of the theory of connections on fiber bundles. After showing how this gauge-theoretic approach organizes and simplifies this problem, we performed explicit calculations to illustrate how a deformable body can, in fact, undergo a net rigid motion by means of cyclic deformations.

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