Numerical investigation of Gravitational Roll Waves

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Abstract

A mathematical model of a gravitational flow of a liquid film above a fixed bed was obtained based on the fundamental Navier-Stokes formulation. It results a pair of non-linear partial differential equations denominated Shallow Waters. This PDE system was discretized with the MacCormack's method[1] and later implemented in FORTRAN. The problem was analyzed in the subcritical and supercritical cases. The results are compared with the Linear Stability Analysis[2] solution and with other analytical studies.

Keywords: Liquid Film, Shallow Waters, MacCormack´s Method.

Introduction

A thin layer of a free-surface flow of an incompressible fluid is usually called **shallow waters state**. By obtaining the Shallow Waters equations, this flow is described in one dimension, in terms of its thickness and average speed. These equations are based on the Navier-Stokes' equations, and they consist of a pair of differential equations that allow to simulate the spatial and temporal behavior of the superficial waves.

Results and Discussion

The basic state used in the simulations was the steady state, which is the basic relation is extracted of the shallow waters equations, when the transients are neglected. Using these condition, the input was applied: a perturbation in the thickness curvature. The results were observed for different time intervals. For a subcritical case, the output is in accordance with the expected results from the Linear Stability Analysis, since occurs the decreasing of the amplitudes with time. But, this theory does not explain accurately the other situation. The presence of interferences (originated by the numerical characteristics of the method) made necessary the use of a numerical filter (Lele´s filter[3]) to smooth the curve.

The shallow waters equations in its conservative form and the results (in graphics) are as follows:

$$\frac{\partial h\bar{u}}{\partial t} = -\frac{\partial h\bar{u}^2}{\partial x} - \frac{g cos\theta}{2}\frac{\partial h^2}{\partial x} - \frac{C_f \bar{u}^2}{2} + gsen\,\theta h + D$$

$$\frac{\partial h}{\partial t} = -\frac{\partial h\bar{u}}{\partial x}$$

D is the approximated diffusive term.

Image 1. Result of a subcritical case.

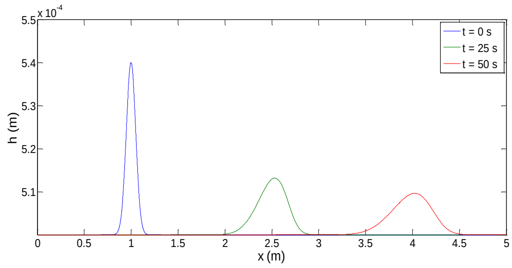
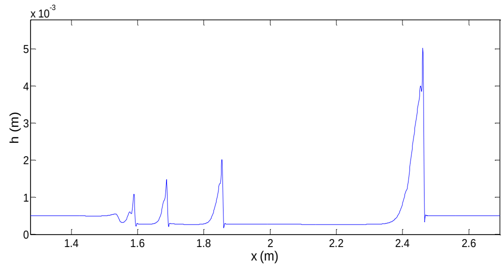


Image 2. Result of a supercritical case making use of the Lele's filter[3].



Conclusions

The MacCormack's method has shown to properly solve the shallow waters problem in its subcritical situations. In the supercritical case, the original methodology amplifies numerical disturbances, requiring the use of containment mechanisms for numerical errors.

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