

## How are trigonometric functions indeed computed?

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### Abstract

In this paper, we study the CORDIC algorithm. Specifically, we study how it can be used to compute trigonometric functions and to calculate the Singular Value Decomposition. In addition, we compare the results obtained using CORDIC with the ones obtained using traditional methods in order to verify its precision.

*Key words: CORDIC, trigonometric functions, Singular Value Decomposition.*

### Introduction

The CORDIC algorithm was developed in 1959 by Jack Volder, who worked for Convair, in order to replace the navigation system of the B-58 bomber.

In time, people realized that CORDIC could be modified and used to solve other problems, such as the Singular Value Decomposition.

In this paper, we study the original CORDIC, which is used to calculate the sine and cosine of different angles, and modify it in order to calculate the Singular Value Decomposition. To verify the precision of the original CORDIC, we compared it with the Taylor Series. In the other hand, to verify the precision of our version of the Singular Value Decomposition, we compared it with MATLAB's version.

### Results and Discussion

To compare the original CORDIC with the Taylor Series, we calculated the sine and the cosine given by both methods and compared it with the value given by MATLAB. In addition, we measured the number of operations each method needed to execute to obtain that result.

As a consequence, we were able to conclude that the CORDIC is faster, since it performs less operations, and that it has a similar precision to the Taylor's method.

The development of the CORDIC algorithm capable of computing the Singular Value Decomposition required the study of the method that inverts matrices by biorthogonalization, which was created by Hestenes.

The precision of the program we developed was verified by comparing our results with the results

given by MATLAB. The Singular Value Decomposition of a matrix is given by  $A = USV^T$ . Consequently, the multiplication of the decomposed matrices is supposed to be equal to the original matrix. Therefore, the difference between the original matrix and the matrix obtained with the multiplication measures the precision of the decomposition. As a consequence, we calculated this difference for several trial matrices using both the MATLAB and our program. As a result, we were able to conclude that our algorithm is very precise for square matrices and for matrices with more rows than columns. However, it was not as precise as MATLAB for matrices with more columns than rows.

### Conclusions

In this paper, we were able to conclude that the original CORDIC is a very precise, fast and economic method. In addition, we verified that it is possible to use a modified CORDIC to compute the Singular Value Decomposition with a good precision for square matrices and matrices with more rows than columns, and a not as good precision for matrices with more columns than rows.

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