

Introductory Topics in Mathematical Analysis and in Algebras.

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Abstract

This project brings a brief overview of the principal components of Mathematical Analysis and of a couple of the first concepts in Abstract Algebras. In the field of Analysis it starts from the axioms of Peano and gradually works its way to characterize the natural, rational and real numbers. Some attention is given to the nature of Series and Sequences. And concludes analysis with comments on functions. In the field of Algebra we explore rings, Ideals and Factorization domains.

Key words: Analysis, Rings, Factorial Domains.

Introduction

The objective of this project is to provide an introductory overview of some topics in Algebra and some in Real Analysis.

The subject of Algebra was chosen as it has many applications in different fields of engineering and even in further studies of mathematics. Therefore, a basis on this topic can be of use to from various specializations. The subject of Analysis, on the other hand, was chosen as its concepts provide the basis for any serious mathematical investigation and therefore is of interest to anyone who wishes to be involved in theoretical research in any field of exact sciences.

Results and Discussion

In the study of Analysis we start by deconstructing the sets of the Real numbers, integers and natural numbers only to reconstruct them with very explicit definitions and pinpointing the particular characteristics that differentiate one of them from another, or what part of these sets lead to a specific characteristic.

As an example, we can explore what differs between the real numbers and the rational numbers. In addition, we have to pass through the notions of sequences and convergence. With them, we can see the real numbers as the set of the limits of all possible sequences in the rational numbers set.

Another important part that is explored is some notions of topology. Open sets, closed sets, compact sets.

Finally, with sets well defined and characterized it is possible to analyze the functions characteristics. It is necessary to have sets well defined to analyze functions because some of the most fundamental theorems in calculus are valid for very specific domain types.

Henceforth we move on to the study of algebra. In it we start by define an object called

Ring. Moreover, it is shown how the set of integers, of real numbers, of $n \times m$ matrixes are all example of rings. We go on to define Ideals and quotient rings and finally end by exploring what characteristics a ring needs in order to have what could be called an unique factorization into primes like in is true for the natural numbers.

Conclusions

In brief we can describe the study of analysis as the building blocks for a strong and stable mathematics. It forbids the use of intuition as mathematical evidence. This is especially important the more abstract is the field in which mathematics is exploring and it helps guaranty that theorems are solid enough to build other theorems on top of.

The study of algebra brings a new level of abstraction to the sets like the integers and matrices, putting them in to more generic classifications and extracting the roots of their properties.

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